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A NEO-FREGEAN (ONTO)LOGICAL FUZZY FRAMEWORK

Lorenzo Peña (*)

INTRODUCTORY REMARKS

This paper puts forward some philosophical considerations in support of setting up and espousing a nonclassical logic (viz. *transitive logic*), which: 1) is infinite-valued, thus recognizing infinitely many degrees of truth; 2) accepts the principle of endorsement (whatever is not wholly false is true; or, in other words, whatever is more or less true is true, period), due to which it keeps the principles of excluded middle and noncontradiction, even though by so doing it entails that some contradictions are true; 3) it isn't ω-overinconsistent, which means that it doesn't rule out the ω-rule.

This paper's title is due to the present approach, as well as the more basic philosophical outlook underlying it, remaining faithful to Frege's most valuable contributions to the philosophy of logic, such as his staunch and uncompromising realism, his conception of numbers as classes of mutually bijectable sets, his reduction of arithmetics to logic, his extensionalistic trend, his very conception of logic as an axiomatized doctrine. As for the formal system whose philosophical motivations I'm here going to back up, it has been put forward in a number of essays listed in the bibliography at the end of this paper.

Sect. 1.- DEFINITENESS, FUZZINESS, AND ASSERTABILITY

Frege held that every concept, being a function, must be definite, i.e. must be such that, for any object whatever, either this object falls under the concept or else it doesn't. Hence, every concept, ϕ, is such that, given an object whatever, x, either $\phi(x)$=Truth, or else $\phi(x)$=Falsehood. For, what could debar a concept from doing so? Nothing since it is a function, and every function is definite. What may alone fail to be definite is a concept-expression, which failure is to be attributed to our failing accurately to determine which concept is denoted thereby. Thus, Frege counts among the legion of philosophers which trace fuzziness back to the subject's failures or limitations.

My first concern in this paper is going to be a criticism of the claim that fuzziness is mind-dependent. I'll endeavour to show that it's far more plausible to regard fuzziness as an objective feature of the world (see esp. below, Sect.3). And it will emerge that fuzziness is compatible with the principles of noncontradiction and excluded middle, even though, at the same time, every fuzzy situation happens to be a contradiction.

What does definiteness mean? Since Frege's death, many attempts have been made to elucidate that intricate matter, and what in Frege's mind was but one feature has been analysed into several different notions: bivalence, two-valuedness, several external and internal, semantic and syntactic principles of excluded middle, and so on. By 'definiteness of a concept ϕ' we may mean one of the following situations:

1º) That, for every object x, either it's truthfully assertable that $\phi(x)$ or else it's truthfully assertable that it isn't the case that $\phi(x)$.

2º) That, for every x, the following disjunction is truthfully assertable: either $\phi(x)$ or it's not the case that $\phi(x)$.

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3º) That, for every x, it's wholly true that either $\phi(x)$ or not $\phi(x)$.

4º) That, for every x, either it's wholly true that $\phi(x)$ or else it's wholly false that $\phi(x)$.

5º) That, for every x, it's, at least to some extent, true that either $\phi(x)$ or not $\phi(x)$.

6º) That, for every x, either it's at least to some extent true that $\phi(x)$ or else it's at least to some extent false that $\phi(x)$.

7º) That, for every x, it's true that either $\phi(x)$ or not $\phi(x)$.

8º) That, for every x, either it's true that $\phi(x)$ or else it's false that $\phi(x)$.

I contend that (3º) is equivalent to (4º), that (5º) is equivalent to (6º), and that (7º) is equivalent to (8º), whereas (1º) is by no means equivalent to (2º). Moreover, while I maintain that every concept is definite in the sense of (2º),(5º),(6º),(7º),(8º), I flatly reject that every concept should be definite either in the sense of (1º) or in the sense of (3º)-(4º).

My reason for deeming (3º) to be equivalent to (4º),(5º) to be equivalent to (6º), and (7º) to be equivalent to (8º) is that the operators 'it's true that', 'it's wholly true that' and 'it's at least to some extent true that' distribute over disjunction. This is obvious for 'it's true that', since -according to Frege himself-'it's true that' is redundant. As for 'wholly' and 'at least to some extent', the ground for maintaining their disjunction-distributivity is that, since they are extent-functors, i.e. extent-modifiers, the result of prefixing one of them to a disjunction is bound to be equivalent to the result of prefixing it to each of the disjuncts, since a disjunction's truth-degree is, in every respect, the greatest among the disjuncts' truth-degrees.

Now, why is (1º) not equivalent to (2º)? For, truthful assertability is not distributive over disjunction. That alone is truthfully assertable which is true in all and every respect -- unless of course we're aiming at some particular respect of reality, but then the sentence meant is the result of prefixing to the one we actually utter the prefix 'in the respect...', or the like. We can conceive of respects as "worlds", provided we regard them as being all encompassed by Reality, i.e. by the real world as a whole. Every world or respect is, so to say, a layer, a region, a sphere of the real world. And only what holds in all respects of the world is truthfully assertable, it alone can be asserted to be true *tout court*. Any alternative policy would lead either to unduly privileging some particular respect (bestowing upon it the monopoly of truth, which, besides being arbitrary, comes on philosophical snags I've analysed in (P:04)) or to being forced to give up the adjunction rule (p , $q \vdash p.q$), or else to denying that there are different respects or "worlds", and so to ignoring Reality's complexity.

Accordingly, since 'It's truthfully assertable that' doesn't distribute over disjunction, all the reasonableness of (2º) nowise carries over to (1º). The rationale for upholding (2º) is that, if we hold (7º), we're just asserting it, and so we're just doing what (2º) says is right, or truthful. But, why do we uphold (7º), i.e. (8º)? For, this much is to be conceded to Frege, since otherwise unsurmountable difficulties would pile up. Furthermore, the main objections levelled against this principle (the simple excluded middle) crumble once a behooving approach is taken -- especially difficulties attendant upon fuzzy sets and situations, as I'm going to show below, in Sect. 2.

Obviously enough, (7º) implies (5º) -- hence (6º) -- since, for any "p", the fact that p implies that it's at least to some extent true that p. What I claim to be wrong is, besides (1º),(3º), as well as its equivalent (4º). What (3º) says is that every substitution-instance of simple excluded-middle (i.e. of (7º)) is wholly true, i.e. entirely true, one hundred percent true. And that is what turns out to be incompatible with the existence of fuzzy situations. What in turn is said by (4º) is that any entity's falling under any given concept whatever is such that either it altogether takes place or else it doesn't take place at all -- since "it's altogether false that p" is equivalent to "it isn't the case that p at all". This is the principle of maximality, or the *Parmenidean* principle of excluded middle: between complete truth and complete falseness there is no intermediary *at all.* Needless to say such tenet is precisely what fuzzy approaches are bound to challenge. For, as I'm going to argue, fuzzy situations hold up to a point only, their holding being partially real and also partially unreal.

Before bringing up this issue, let me point to an important instance of (7º), viz.: (9º) For every x, (it's true that) either it's at least to some extent true that $\phi(x)$, or else it isn't true that $\phi(x)$ to any extent.

(9º) is the weak principle of excluded middle. (Of course 'it's not the case that it's at least to some extent true that' is equivalent to 'it isn't true that... to any extent', which in turn boils down to 'it's not at all the case that'.) What is important about (9º) is that it's weaker than (7º), and many systems which countenance (9º) don't countenance (7º); such are, e.g., Gödel's logics as well as Łukasiewicz' logics enriched with a functor meaning 'at least to some extent' -- a functor 'L' such that $/Lp/=1$ iff $/p/\neq 0$, and else $/Lp/=0=/p/$. Still, (7º) can be derived from (9º) by way of the *endorsement rule*, namely: $Lp \vdash p$ (from "It's at least to some extent the case that p" to infer "p").

Sect. 2.- DEGREES OF TRUTH AND THE PRINCIPLE OF EXCLUDED MIDDLE

What Frege intended to reject is that there might be cases wherein some entity neither falls under a given concept nor fails to fall under it. Yet, there are such situations, as have been contended by all proponents of fuzzy-set theories, and as was shown by Plato and the Megarics, and more recently by many philosophers, e.g. Engels (remember Engels' examples: a dialect which neither belongs nor fails to belong to some given family of dialects, a species that neither is included nor fails to be included in some genus; see Engels' *Dialectik der Natur* and my own analysis thereof in (P:02), pp. 16-8; still more interestingly, Engels openly espoused the view that the boundary between truth and falsity is fuzzy: see his *Ludwig Feuerbach* and again (P:02)).

The rationale for espousing the degrees-of-truth claim encompasses, among others, the following points.

1.- Since any state of affairs is a thing or entity, and since its truth is nothing else but its existence, whenever a state of affairs is truer than another, the former is more real. Now, there are many degrees of sluggishness, sadness, strength, cowardice, greatness, smallness, and what not. But, if, e.g. Tom is more wicked than Ted, then Tom's wickedness is truer (more real or existent) than Ted's wickedness. For, 'Tom is more wicked than Ted' abbreviates 'It's truer that Tom is wicked than that Ted is wicked'. (Any other account of comparatives I know of seems to me either farfetched or unwieldy.) There's nothing circular here, since "It's truer that p than that q" abbreviates "The fact that p implies that q, while it's nowise the case that the fact q implies that p", implication being a relation between facts such that "p implies q" says the same as "That both p and q is equivalent to that p". (I'm bound to stress that the amazingly huge fruitfulness of fuzzy approaches, not just in the social sciences but in sciences of nature as well, shows that most sets science is concerned with are fuzzy rather than crisp.)

2.- While there are (to be sure debatable) grounds for holding the reality of sets or classes, some classes seem to have a stronger claim on their reality than others. Now, if we deem all of them real, a stronger claim on reality seems to mean nothing else but a claim on greater or truer reality. (What I'm supposing is that whenever it is more natural to speak of some class's reality than of some other's, the former is, *caeteris paribus*, more existent than the latter.) Other things being equal, a class, x, seems to be more real than another, y, whenever either (i) it's truer that something or other belongs to x than that something or other belongs to y (which is why the man in the street loathes talk about entirely empty sets; there's a strong rationale for that, since universals -- that is, sets -- exist in, with, and through their members); or (ii) such entities as are somewhat belonging to x are more alike than the ones somewhat belonging to y.
3.- The methodological principle just stated for classes applies to states of affairs, too: whenever it's more natural to speak about some fact or event's reality than about some other's, the former is truer (i.e. more real). Now, there may be a number of reasons for it to be more, instead of less, natural to speak about some fact's reality: its longer duration, its greater causal impact, or -- perhaps even more important -- its normality or typicalness within its immediate environment -- a human individual's, or a human collectivity's, action's immediate environment being the set of actions done by the same subject.
4.- It's a moot question whether there are value conflicts. Anyway, there being conflicts between opposite values and duties is most easily and naturally explained in virtue of value-fuzziness: there are worthiness-degrees (see (P:12)). That means that some value-statements are truer than others; hence, some true value-facts are truer (more real) than others. (Casuistics of course vainly tries to eschew duty-contradictions by conditionalizing duty statements with more and more complicated protases. Unrewarding as such efforts are, taken at their face-value, there may be some point in them, insomuch as what is -- or, more accurately, should be -- at issue is to settle on which conditions some duty is to be given precedence over duties opposite thereto, without thereby cancelling them.)
5.- As Zeno's paradox of the arrow shows, movement is contradictory. Like any other true or real contradiction, movement's contradictions are ensuant upon fuzziness -- through the rule of endorsement. What movement's contradictions show is that, at any time-interval comprised within the movement-duration, the travelling body both lies (to some extent or other) in any given stretch included in the whole movement-span, and in some degree or other, too, doesn't lie therein. Simply admitting the contradictions without admitting infinitely many degrees of reality wouldn't do, since it's quite unbelievable that at every such time-interval the body lies in *all* such stretches to the same extent. (Speed would then become incomprehensible.) There must be a difference of degree. And since there are uncountably many such stretches, there are bound to be uncountably many degrees of truth.
6.- Evolution furnishes us with a strong reason for accepting fuzziness, and hence truth-degrees. Ichthyostegas, while somewhat exemplifying the property of being a fish, did somewhat exemplify the property of being a quadruped, too. So, an ichthyostega's being a fish is real or true, but surely less true than a herring's being a fish.

The foregoing remarks' outcome is that not only are there grounds for recognizing the existence of many degrees of truth or reality, but moreover we find in ordinary ways of thinking hints for useful, if incomplete, criteria on degrees of truth. This is important, since a major objection against talk about degrees of truth or reality is the claim that that talk is bound to be false or at least idle unless we have truth-degree criteria. While such a claim is mistaken, it's still true that only having such criteria gives the theory useful applications, thus allowing it to become integrated within our general theoretical framework.

Whenever we encounter a fuzzy situation what happens is not that we "don't know what to say", but that we say that it neither takes place nor fails to take place. Now, in virtue of involutivity (of simple negation, the mere 'not') and De Morgan, "It neither is nor fails to be the case that p" is equivalent to "It is and it isn't the case that p", i.e. a contradiction. Therefore, countenancing fuzziness is bound to lead to countenancing contradictory truths.

Some people say that fuzzy situations are not such situations as neither hold nor fail to hold, but such situations as we cannot say that they hold and yet we cannot say that they don't hold either. According to that approach, a fuzzy situation is a situation whose corresponding excluded-middle instance "fails", not in the sense of being false, but in the sense of being indeterminate. However -- they go on to say -- that indeterminateness doesn't consist in its failing to be either true or not true -- since then we should again, due to redundantness of 'it's true that', as well as involutivity and De Morgan, come back to a contradiction --, but in their being such that nothing can be said about it, neither that it holds nor that it doesn't hold, nor that it neither holds nor fails to hold, nor that it either holds of else fails to hold, nor that it is not such that it both holds and fails to hold, nor of course that it both holds and fails to hold. (My own approach instead contends that a fuzzy situation is one which neither holds nor fails to hold, and so both holds and yet doesn't hold.) That standard approach to fuzziness is open to these objections:

1) It plunges us into ineffability, since -- at least until other functors are introduced -- nothing could be say about a fuzzy situation

2) It drives a wedge between its being the case that p and our being able to say that p ('being able' in an objective sense: being such that, if we say that p, we are right, i.e. we say something true); for, according to that view, even though its being the case that p is fuzzy -- and so ineffable -- our being able to say that p is flatly false and so unfuzzy.

3) It compels us to jettison the principles of noncontradiction and excluded middle, which results in a saddening and untoward weakening of the system of logic accepted.

4) It runs counter to the linguistic evidence about what people actually say as regards fuzzy situations: instead of holding their tongues, they say 'neither yes nor no' or 'yes and no'.

5) Unless they countenance truth-value gaps -- which are irksome, to say the least -- their proponents are bound to assume truth-values different from (whole) falseness which still are not truth-*degrees*. Now, any many-valuedness conceived of in any other way than multiplicity of truth-degrees is hard to accept: degrees of nearness to truth, e.g., are difficult, if not impossible, to accept unless one is prepared to admit degrees of truth (for what "p"'s *being nearer the truth than* "q" amounts to, if it doesn't amount to its being objectively truer that "p" is near the truth than that "q" is near the truth? But then, we are stranded again on truth-degrees, which we had just tried to replace with de-

grees of proximity to truth).

6) It mistakes being wholly true (or anyway reaching some truth-threshold above 50%) for being true (*tout court*). That seems wrong for several reasons. For one thing, as being tall is not the same as being completely tall (who is?), likewise being true is by no means the same as being altogether true; the comparison seems warranted by the similar features shared by those properties -- and by the expressions denoting them --. For another, it forbids equating 'Bernard is smarter than Graham' with 'It's truer that Bernard is smart that that Graham is smart', which equation, besides being highly plausible, offers a winsome, rewarding and straightforward account of comparative constructions. Finally, it allows the equiparancy rule: p, q ⊢ It's as true that p as that q; which rule seems to me blatantly wrong, as can be shown by means of a great many examples.

Consequently, I reject not just the classical approach, according to which there are not fuzzy situations *at all*, and the Parmenidean principle of excluded middle holds, but also the standard nonclassical approach to fuzziness, according to which the (simple) principle of excluded middle fails for those situations.

Sect.3.- FUZZINESS vs DOUBTFULNESS

Most people interested in the vagueness issue have somehow or other conflated two different questions: objective fuzziness with subjective inexactness (or imprecision, or doubtfulness, or likeliness, or something of that ilk). That a set lacks any clear criterion of membership nowise makes the set fuzzy-unless, of course, we embrace verificationism; only once verificationism has been espoused, the very issue of whether there may be *objective* fuzziness vanishes altogether. Nor are we bound to regard a set as fuzzy simply because it is easier for us to ascribe to several things different degrees of membership thereof unless we're prepared to say those are objective degrees, not merely degrees of probability.

What has in no small measure lead to so a muddling confusion is talk about fuzzy or vague *concepts* -- thought of in the darkly half-psychologistic way sturdily denounced by Frege --, which we are more prepared to apply to some entities, less so to others. Now, our being prepared in degree u to apply concept x to entity z may be construed either literally or as our being prepared to apply x to z in degree u. In the first case we are dealing with some sort of probability, plausibility, verisimilitude, or some other kind of likeliness in a wide sense. In the second case, instead, we deal with genuine objective fuzziness, or at the very least with a subjective belief in objective fuzziness. Of course there are connections between them. Very often our being more inclined to ascribe a property to a thing than to another can be easily grounded on -- and explained through -- the former thing's exemplifying the property to a greater extent than the other's. Nontheless, likelihood -- and akin notions -- is a matter different from fuzziness or objective graduality.

What is at issue when we encounter vague "concepts" or "representations" is whether they stand for vague or fuzzy properties. (Of course the problem would vanish were we to espouse thoroughgoing nominalism.) Likewise, when we're inclined to ascribe in a degree u a property x to an entity z, the question arises of whether z's exemplifying x is real in a degree u. Suppose, e.g., that 'tall' is a vague dummy which we're compelled to use only for commodity sake or for lack of any better replacing term. Then, if we agree that there are facts, the issue arises of whether there are facts such as Brian s being tall. To be sure these questions can be shunned if we're keen on eschewing onto-logical problems altogether, or if we uphold a radically nominalistic view on sets as well as on facts. Still we're then lead to identify not only doubtfulness (or any germane notion) with fuzziness but also certainty with truth, which seems to me unwise. Anyway, such tenets run counter to the healthy Fregean realism which so carefully differentiates subjective attitudes from objective truth.

This is why choosing a gradualistic approach is not a matter of mere epistemic convenience. Beyond such kind of considerations lies the fundamental question: does our epistemic convenience agree with how reality is and what entities, properties and facts it does encompass or contain?

What the fuzziness-doubtfulness confusers can be charged with is what Hegel pinned down as a basic mistake in Kant's dialectic of pure reason: a tenderness towards reality debarring him from ascribing contradictions to it. Likewise, like Kant and many other people, those confusers seem to think that, once a difficulty has been confined to the mind's realm, it has been straightened out; thus, whenever a problematic kind of entities is encountered, it is purportedly disposed of by way of being reduced to something inside the mind. But, why what is perplexing or cumbersome or difficulty-ridden will become unproblematic simply by being regarded as something inside the mind?

In this light we can fruitfully approach the question of whether vague predicates can and/or must be eliminated. Are we able to replace fuzzy predicates with crisp ones *and yet say the same things?* Surely not, if the things formerly said were fuzzy facts, facts whose existence was true only up to a point. But are we able, by resorting to the replacement under consideration, to say something which is sufficiently akin to what was formerly intended to be said? Well, yes -- although the question is of course pretty vague itself: we can in every case say something which, while being crisp, necessarily entails what we previously said, or tried to say. To put it more precisely: we could achieve that result were we able to make the replacement; and we could make it could we secure a sufficiently wide knowledge; which is a *practical* impossibility. Now the most important question to be answered on this score is: why is that attainment practically impossible? In other words: why is then pragmatically more convenient to cleave to fuzzy predicates? From a realistic viewpoint the answer is quite straightforward: because fuzzy sets (or properties) and facts do exist, and one same fuzzy fact may be entailed by many different crisp ones incompatible with one another (e.g. being young is entailed by being-exactly-7301-days-and-two-hours-and-four-seconds-old); hence, it is easier to become aware of a fuzzy fact. Of course we can have disjunctive crisp predicates like being-either-eighteen-years-old-or... Such predicates will be biconditionally linked with the fuzzy predicates they would be called to replace -- and thus would be indiscernible thereof according to antigradualistic standards, which recognize no equivalence stronger than the mere biconditional. Only, nothing could exemplify one such predicate more than any other thing; thus comparative constructions would lose any point or even become illicit, ill-formed. Construed like this (as a disjunction of crisp predicates) youngness doesn't come in degrees, and two youths whatever are as young the one as the other. Moreover, there arise difficulties over how those disjunctive crisp predicates are generated and arrived at -- usually we see that something is red-or-violet either by seeing it's red or by seeing it's violet; thus, epistemically at least, the union of two predicates seems to be dependent and attendant on those predicates; while its fuzzy counterpart needn't be so. Furthermore, any such crisp disjunctive predicate is bound to be made up by infinitely many disjuncts, which is anyway unfeasable in finitary languages, like natural languages and most formal languages.

My conclusion is that the elimination of fuzzy predicates is: 1) practically unfeasible (which gives rise to the question: *why* does this happen?); 2) theoretically undesirable *if* there are fuzzy sets of fuzzy facts (which I for one hold to be real --up to a point of course!); 3) anyway leading to the abandonment of comparative constructions, which are so deeply rooted in all our world-view and way of speaking.

The kind of remarks just advanced may help to understand what factual links there are between doubtfulness and fuzziness. Those links ought to be acknowledged, precisely in order to prevent the confusion between both notions. When I've ascertained that a house is big, it may remain doubtful to me exactly how big it is. Thus, if I have ascertained that the house exemplifies the crisp disjunctive counterpart of bigness, I may have still not ascertained which one of the infinitely many predicates whose union is the counterpart in question is exemplified by the house. (Notice though that my ascertaining the house's exemplifying the crisp counterpart of bigness in this case happens to be ensuant upon my having ascertained the house's being big; it's not bigness' crisp disjunctive counterpart's being exemplified by the house what is ascertained of and by itself.) Therefore, such doubtfulness as goes with fuzziness is doubtfulness not about what things belong to the fuzzy set in question, but about to which crisp set does each of those things belong among the mutually opposite crisp sets membership of which entails membership of the fuzzy class under consideration. Yet, a property's fuzziness doesn't lie in such doubtfulness, which turns out to be attendant upon it, but in the fact that there are some entities which exemplify the property up to a point only, and therefore also fail -- to some extent or other -- to exemplify it.

Sect.4.- THRESHOLDHOOD AND KINDS OF ω-INCONSISTENCY

My approach emphasizes infinite-valuedness, but, unlike most logics of fuzziness, it postulates thresholdhood, namely that, for any fact, that p, there is a lower threshold of p, viz. np (its being overtrue that p) as well as an upper threshold of p, viz. mp (its being much like true that p). Notice that a fact may be identical either to its lower or to its upper threshold. Its acknowledging thresholdhood sets this approach off as a *transitive logic*, a logic of *transitions*. Thus, our approach establishes that the scalar truth-values form a set which is *quasi dense* (if u>v, u and v being scalar truth-values, either mv=u, or nu=v, or else there are infinitely many values greater than v but lower than u) and yet also strongly *pseudoatomic*, in this sense: for any value u there are two values -- such that one of them is at least as true as u, while the other is at most as true as u -- such that there is at most one value between them. As for the minimum value, 0, it can have no lower threshold different from itself; thus n0=0. But has it an upper threshold m0≠0? Or, which amounts to the same, has 1=N0 (where 'N' stands for simple negation) a lower threshold n1≠1? My contention is that they do: m0≠0 and Nm0=n1≠1. Otherwise we'd be stranded on an extremely obnoxious ω-overinconsistency. Let's first distinguish several kinds of ω-inconsistency. ('E' will be the existential-quantifier prefix, while 'U' will be the universal-quantifier prefix.)

A system S is ω-inconsistent for a negation '∿' iff it contains some formula "p|x|" such that even though, for every individual symbol, 'z', "p|x/z|" is a theorem of S, still "Ex∿p" is also a theorem of S. A system S is strongly ω-inconsistent iff for some formula "p", even though, for every individual symbol 'z', "p|x/z|" would trivialize the system -- render it deliquescent, i.e. such that every wff is a theorem -- were it added to S's axioms, still "Exp" is a theorem of S. A system S is ω-overinconsistent iff for some formula "p", even though, for every individual symbol 'z', "p|x/z|" is a theorem of S, adding to S's axioms "Uxp" would trivialize it. A system S is irremediably ω-overinconsistent iff every stark extension of S (i.e. every extension of S that keeps all inference-rules of S) is ω-overinconsistent. A system S is sturdy iff it has no conservative irremediably ω-overinconsistent extension. Our formal system is strongly ω-inconsistent, besides being ω-inconsistent for the simple negation 'N' (the mere 'not'). But it is sturdy. Instead, no system taking as designated truth-values all standard reals r such that 0<r≦1 is sturdy; suffice it, in order to prove it, to add a formula (e.g. "x is near the Sun"), "p", every substitution-instance whereof would be a theorem and there would be an infinite descending chain of the values of "p|x/z|" for different symbols 'z', having as its limit 0; however, since, for every r≧0, the g.l.b., or infimum, of the standard real interval]0,r[is 0, we'd have that /Uxp/=0.

What is wrong with irremediably ω-overinconsistent systems is that they *necessarily* lack the ω-rule (obvious from the definition) while strongly ω-inconsistent systems can have the ω-rule. Now, of little practical use as it may be, the ω-rule is of itself correct for any language sufficiently rich; accordingly nothing must stand in its way, that is to say: a well behaved system must contain nothing hindering further introduction of that rule -- in a sufficiently rich extension of the system. What is unpalatable with ω-overinconsistency is then that it allows an absurd situation like this: every entity taken apart would satisfy some predicable, and yet it would be absolutely false, trivializing, that all things satisfy that predicable. What would generate the absolute falseness would be the mere "putting together" all things instead of leaving them apart fron one another.

In order to escape ω-overinconsistency we need: either 1) to reduce the set of designated scalar truth-values to some proper subset of]0,1] , which would counter to the endorsement rule (and, moreover, wouldn't be sufficient to prevent some forms of quasi-ω-overinconsistency, like, e.g., having a system S with a formula "p|x|" such that even though, for every individual symbol 'z', adding to S's axioms "p|x/z|" wouldn't trivialize S, still adding thereto "Uxp" would -- and such a dreary fate would befall even some alethically maximalistic systems, i.e. systems with just value 1 designated); or else 2) to postulate a (scalar) *designated* value which should be the g.l.b. of the set of (scalar) values greater than 0. This is what our system of transitive logic does.

A system of the sort I've been arguing for may be set up like this: We take the (standard) real interval [0,1]. For any real r such that 0≦r≦1, let the three following pairs be called *hyperreals:* {r,2}, {r,3}, {r,4}. We now introduce an order ≦ like this: if r<r´, then {r,2} ≦ {r,3} ≦ {r,4} ≦ {r´,2} ≦ {r´,3} ≦ {r´,4}. Let such hyperreals h as {0,3} ≦ h ≦ {1,3} be called *alethic elements*. We define operations: if r(0≦r<1) εh, then mh={r,4}; m{1,3} = {1,3}; m{1,2} = {1,2}; if rεh and nεh (where n=2, 3 or 4), then Nh = {r´,n´}, where $r´=2^{\log_r 2}$, while n´=2 if n=4, n´=4 if n=2, and n´=3 if n=3; hIh´={½,3} if h=h´, otherwise hIh´={0,3}; Hh={1,3} if h={1,3}, otherwise Hh={0,3}. Let *D* be set of alethic elements different from {0,3}. We now define our formal system as <*V*,*F*,*T*>, where *V* = {m,I,↓,H}; *F* is defined like this: whenever p,q ε*F*, so do pIq, p↓q, Hp, mp; let v be any function from *F* into the set of alethic elements; then v is a valuation iff for every p,q ε*F*, v(mp)=mv(p); v(pIq)=v(p)Iv(q); v(p↓q) = *min*(Nv(p),Nv(q)); v(Hp) = Hvp; then *T* is {pεF: every valuation v is such that v(p)εD}. The intended meanings are: pIq: It's true that p to the same extent as that q; p↓q: neither p nor q; Hp: It's wholly true taht p; mp: It's much like true that p.

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